NOTATION

C_x, drag coefficient for a particle; D_p, particle diameter; g_i, components of the acceleration g due to gravity acting on a particle in the direction of jet flow (g_i = g sin α) and in the direction normal to it (g_i = g-cos α); $V'^{\pm}_{p0i}$, $V'^{\pm}_{g0i}$, fluctuation components of the velocities of the particles and gas, respectively, at the end of a mole formation; V_{fi}, free-fall velocity of a particle; l_u, mixing length; m_p, particle mass; t_p, length of time of particle–mole interaction; $V'^{\pm}_{pi}$, $V'^{\pm}_{gi}$, positive and negative fluctuation velocities of particles and of the gas respectively, with the components $u'^{\pm}_p$, $u'^{\pm}_g$, $v'^{\pm}_p$, $v'^{\pm}_g$, $k = V'_{g0i}/V_{fi}$; $V'^{\pm}_{\sim i}$, relative velocity of the gas; α, jet inclination angle relative to the earth's surface; β, empirical constant; δ_u, $\delta_\varkappa$, jet boundaries in terms of velocity and concentration, respectively; $\eta_u = y/\delta_u$, dimensionless velocity ordinate; $\eta_\varkappa = y/\delta_\varkappa$, dimensionless concentration ordinate; $\varkappa$, admixture concentration; u_m, $\varkappa_m$, velocity and the concentration of the admixture at the jet axis, respectively; μ_g, dynamic viscosity of the gas; ρ_s, ρ_g, densities of the particle material and of the gas, respectively; τ_g, τ_p, shearing stresses in the gas and in the "gas" of particles, respectively; and τ_m, τ_0, shearing stresses in the mixture and in pure gas, respectively.

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NUMERICAL SOLUTION OF THE FORWARD ONE-DIMENSIONAL PROBLEM OF CRITICAL FLOW IN NOZZLES

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The principles of an algorithm are formulated for a numerical solution of problems of one-dimensional flow in nozzles with passage through a singularity. The results of calculations are compared with experimental data.

In engineering practice one often encounters problems involving the calculation of flow parameters for channels of variable cross section. In the simplest case this would be the flow of an ideal gas without friction and heat transfer. More complex problems include those involving the flow of a real gas with friction at the walls and with heat transfer, with expansion of two-phase or multiphase media accompanied by interphase interactions, with motion of multicomponent mixtures accompanied by chemical reactions, etc.

According to an earlier study [1], all these problems can be classified into forward and reverse ones. The latter are widely encountered in numerical analysis of the motion of various media through nozzles, inasmuch as here the solution does not have a singularity. In the analysis of flow through channels of a given geometry (so-called forward problem) there arise difficulties due to the fact that the solution contains a singularity of the saddle kind. It is not possible to obtain a continuous solution for critical flow and, therefore, special methods are used allowing the singularity to be taken out. Such methods are replacement of the steady-state problem with the transient one (method of stabilization), replacement of the forward problem in the vicinity of the critical point with the reverse one, and other methods. Such approaches have already been thoroughly surveyed [1, 2].

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Here a method will be described which yields a continuous solution to the forward problem for the critical flow of a medium.

The simplest case with an already known analytical solution has been selected as the model (test case), since both the problem itself and the algorithm of its solution retain their character whether simple cases of the flow of an ideal gas or the flow of media with intricate compositions and with interactions of phases is considered.

The steady motion of an ideal gas without friction through a channel of variable cross section is described by a system of equations where the equation of motion can be expressed as a differential relation [3] between the relative velocity $\lambda = w/\sigma_{cr}$ and the area of the channel cross section F(z)

$$\frac{1}{\lambda}\frac{d\lambda}{dz} = \frac{1}{F(z)}\frac{\frac{k-1}{k+1}\lambda^2-1}{\lambda^2-1}\frac{dF}{dz}. \tag{1}$$

The results of numerical integration of Eq. (1) by the methods of Milne, Adams, Runge–Kutta, Hemming, and Euler respectively have been compared [3] with its known exact solution. The problem was solved on a model BÉSM-6 high-speed computer, with the critical mode of flow extracted by the "shooting" method. The solutions were classified as follows:

$$\lambda \geqslant 1 \quad \text{at} \quad z < z_{min}, \tag{2}$$

$$\lambda < 1 \quad \text{at} \quad z_0 \leqslant z \leqslant L, \tag{3}$$

$$\lambda \geqslant 1 \quad \text{at} \quad z_{min} \leqslant z \leqslant L. \tag{4}$$

Condition (2) corresponds to solutions which have no physical significance, since it is well known that transition through the sound barrier cannot possibly take place in a narrowing channel. Condition (3) governs subsonic flow throughout the entire channel length, and condition (4) corresponds to critical flow of a medium in a DeLaval nozzle.

Values for the initial velocity λ_0 were tried until the equalities $\lambda^2 - 1 = 0$ and dF/dz = 0 would not be satisfied simultaneously between cross sections $F_i \le F_{min} \le F_{i+1}$. With conditions (2)-(4) satisfied, moreover, only the Euler method yields the entire integral curve. The integration formulas according to Milne, Adams, Runge–Kutta, and Hemming yield large rounding errors and oscillating solutions near the critical point. Oscillating solutions make it impossible to select the integral curve on the basis of conditions (2)-(4) even in such a simple case. This explains why all attempts to use the Runge–Kutta method for obtaining a continuous solution to the forward problem have so far been unsuccessful [1, 2].

Satisfactory results can be obtained by the Euler method, even though it is less accurate than the Runge–Kutta and other methods. Its algorithm of the solution is constructed so that $\lambda_{cr} = 1$ is always (boundary condition) at the critical point. The only source of inaccuracy here is the indeterminacy of the location of the critical point within the computation interval and, accordingly, the error of calculation is maximum far from the singularity, i.e., at the channel entrance and exit sections. In our calculations with a 0.01 step this error did not exceed $4\cdot10^{-2}$-$4\cdot10^{-3}$%.

As the problem becomes more complex the fundamental system of equations will be more unwieldy and contain a larger number of differential equations.

The solution of such problems can be approached in two ways. Both involve transformation of all equations describing the process to differential ones. The resulting system of equations will be linear with respect to the derivatives of the thermodynamic quantities and the latter can be easily determined. The solution of this system is then based on methods of numerical integration for determining all thermodynamic quantities at the successive sections in the first case and only a part of them in the second case. The number of functions to be determined from the derivatives must, moreover, not exceed the number of the initial differential equations. The remaining unknowns are calculated by any iteration method applicable to a system of nonlinear algebraic equations (equations of coupling). It is quite obvious that the latter method entails a completely conservative scheme, while the scheme according to the first method cannot be regarded as a conservative one on account of the errors building up in the computation process [4].

For constructing an algorithm according to which more complex problems can be solved, we transform conditions (2)-(4) to

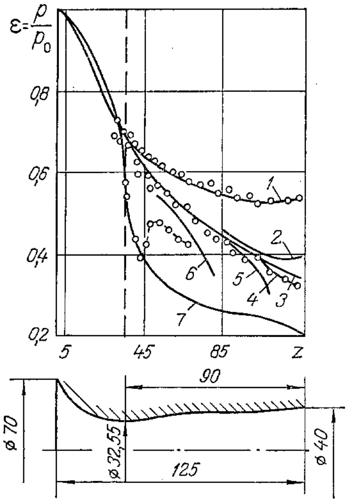

Fig. 1. Pressure distribution of the nozzle length: 1-6) $x_0 = 0.3$; 1) $\xi_0 = 11.68\ \mu m$; 2) $\xi_0 = 10.48\ \mu m$; 3) $\xi_0 = 10.43\ \mu m$; 4) $\xi_0 = 10.42\ \mu m$; 5) $\xi_0 = 10.38\ \mu m$; 6) $\xi_0 = 9.97\ \mu m$; 7) $x_0 = 0.78$ and $\xi_0 = 82.01\ \mu m$; solid lines) calculations; dots) experimental data [9].

$$\begin{cases} \mathrm{sign}(\det_i) \neq \mathrm{sign}(\det_{i-1}), \\ \mathrm{sign}(\Delta_{i,j}) = \mathrm{sign}(\Delta_{i-1,j}); \end{cases} \quad (2a)$$

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Here det denotes the principal determinant of the system of differential equations, its elements being the coefficients of the derivatives of the unknown functions, and Δ_j denotes the determinant which results from a replacement of the column of coefficients of the derivatives of the j-th quantity with the column of free terms. Subscript i refers to the computation step.

The conditions in their new form (2a)-(4a) are valid for problems of any degree of complexity. Condition (4a) is a universal one, moreover, inasmuch as it accounts for passage through the singularity of the solution and thus makes determining the location of the critical section as well as the values of the critical quantities possible.

We note here that combining the Runge–Kutta method and the Euler method for follow-through calculations where the solution has saddle-point singularities is difficult, because in more intricate problems the location of a singularity is not a priori known.

The effectiveness of the proposed methods of solution was checked on the earlier model problem. It has been discovered that, unless the conditions of conservatism are satisfied, none of the methods of numerical integration, including the Euler method, will yield the parameters of critical flow. As to the second (composite) method, it yields exactly the same result which has been obtained by solving Eq. (1). It is to be noted that this composite method contains a certain indeterminacy with regard to the selection of parameters whose values are calculated by numerical integration. The accuracy and the stability of the solution depend on how successfully those parameters have been selected. It is difficult to give beforehand any recommendations facilitating this selection, all depends on the kind of equations formulating the problem and on the experience of the person making the calculations.

Noteworthy is still another feature of the proposed method of calculation, a feature which distinguishes it from other methods described in the technical literature. As the form of the conditions (2a)-(4a) indicates, they are based entirely on the system of equations which describes the flow process. Their use does not require any additional stipulations about the velocity of sound in the given medium. The principal determinant det of the system is a quantity proportional to $(M^2 - 1)$ in the case of homogeneous media and can be tentatively put in the same form also for heterogeneous media [5]. When the Mach number (equal to the ratio of the velocity of the carrier medium to some acoustic velocity) is introduced according to any physical theory which defines that velocity, then conditions (4a) can become inconsistent and no solution to the problem will in this case be obtained. It is a simultaneous change of the signs of all quantities in the determinants which serves as a universal indicator of passage of the solution of the system of equations through its singularity.

The principles which have been outlined here can serve as the basis of an algorithm of solution for various problems such as expansion of an ideal or real gas, flow of a suspension of solid particles in an ideal gas [6], or motion of a vapor-liquid medium through variable-section channels [7, 8].

We show here the solution to the problem of flow of disperse wet steam through a DeLaval nozzle. The system of equations is in this case more unwieldy and contains 42 equations, four of them differential ones. It consists of laws of conservation applied to the mixture as a whole as well as to the individual phases, conditions of heterogeneous equilibrium, and relations characterizing the thermodynamic properties and the transfer properties under limiting conditions and away from them. The results of calculations shown in Fig. 1 are compared here with those of an earlier study [9]. Curves 1-4 depict the modes of subcritical flow which satisfy condition (3a). Curve 7 depicts the critical flow when the critical section coincides with the narrowest nozzle section (indicated by a dash line). The discrepancy between values according to the theoretical curve 7 and experimental data for the expanding nozzle segment can be explained by spontaneous phase transitions observed in experiments but not included in the mathematical description. The bifurcation of the theoretical curve for the contracting nozzle segment corresponds to different initial vapor mass fractions in the stream – $x_0 = 0.3$ corresponds to the right-hand branch and $x_0 = 0.78$ corresponds to the left-hand branch.

Finally, curves 5 and 6 depict modes of flow without physical significance for the given nozzle according to conditions (2a). All the curves in Fig. 1 have been plotted with the input data fully conforming to the experimental conditions. The mean dimension of water droplets ξ_0 at the nozzle entrance was the varied parameter.

This example demonstrates not only that forward problems of critical flow in nozzles can, in principle, be solved but also that even the simplified one-dimensional formulation of such a problem will yield a close agreement between calculated and measured values.

NOTATION

p, pressure; w, velocity; F, area; a, acoustic velocity; ξ, mean radius of droplets; M, Mach number; k, adiabatic exponent; λ, dimensionless velocity; L, channel length; z, longitudinal coordinate; x, true vapor mass fraction. Subscripts: 0, entrance section; cr, critical section; min, the narrowest section; and L, exit section.

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